Effective Field Theory for Light in Disordered Atomic Medium

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Abstract

We develop a field theory approach to light propagation in a gas of resonant atoms taking into account vector character of light and atom-atom interactions. Within this approach, we calculate the propagator of the electric field for both short and long-range density-density correlation functions of the gas.

I.   INTRODUCTION

Transport of waves through strongly scattering disordered media has received much attention during past years. The character of this process is determined by the multiply elastic scattering of light. In a sample of randomly distributed scatterers, the initial direction of the wave is fully randomized by multiple scattering, and a diffusion picture seems to be an appropriate description of light propagation in the case of small densities of scatters [1]. Accordance to this picture, he transmission coefficient of a medium is inversely proportional to the sample thickness, i.e. follows the familiar Ohm's law. Despite successful predictions, this theory does not take into account interference effects. P.W. Anderson was the first to demonstrate that sufficiently strong disorder can result in a localization of quantum-mechanical wave functions [2]. This phenomenon is referred to as Anderson localization. In fact, even away from the localized phase, in the so-called weak localization regime, wave interference affects the physics of light propagation. For example, the constructive interference of counter-propagating light amplitudes results in the coherent backscattering peak [3, 4]. In the past 20 years, mesoscopic physics has been developed, with beautiful experimental and theoretical results pertaining to the weak localization regime [5, 6, 7].

A system of randomly distributed atoms with an atomic transition frequency close to the light frequency, provides necessary conditions for experimental observation of backscattering [8, 9, 10, 11]. In fact, under these conditions, light scattering on an individual atom is characterized by a large cross-section. However, in contrast to classical scatters, in this system one has to take into account resonance dipole-dipole interactions that alter the character of the light propagation. There are two different mechanisms of excitation transfer. According to the first mechanism, an excited atom decays into the ground state by emitting a photon. Then the photon propagating through a system of resonance scatters, is absorbed by another atom, and thus excitation transfer takes place. In the second mechanism, an excitation may transfer from atom to atom nonradiatively by the resonance dipole-dipole interaction. This is equivalent to including the longitudinal component of the electromagnetic field. This mechanism becomes more important with increasing the density of atoms n. If the average number of atoms per cube with the size of the wavelength λ exceeds unity, $n\lambda^3 \gtrsim 1$, the typical energy of dipole-dipole interactions is comparable with the inverse of the the radiative lifetime of excited atoms, and the transport of nonradiative photons dom-

inates the transport of radiative ones. The resonance dipole-dipole interaction also plays an important role in the radiative transfer by binary collisions of atoms in gases[12, 13].

This problem has been actively studied under in the case of low atom number densities[10, 14, 15]. By using the scalar model of light it has been demonstrated that interference effects are substantially suppressed as a result of the motion of scatters[16]. In Refs. [11] the authors considered the vector character of light and took into account atom-atom interaction. It was found that close to the resonance, the transport slows down considerably because the light is captured by atoms for a longer time. Such a behavior results in a considerable reduction of the speed of light. This effect have been observed for the classical systems, but for the dipolar gas the effect differs[8, 9].

In this paper we focus our attention on the case $n\lambda^3 \ll 1$ and consider different density-density correlation functions for light scatters. The latter will be treated as a $(1 + 3)$-level atom (In other words, we consider atoms with a resonant $s-p$ transition.). We also take into account the effects of dipolar atom-atom interactions on the light propagation. The influence of the atomic density-density correlation function (the second cumulant, or second central moment of the probability distribution for different atomic realizations) is of special interest for us. It appears that in the considered case of a low density system, only this correlation function affects the distribution of total transmission coefficient. We therefore assume that the other cumulants are zero (that is we consider gaussian probability distributions). This means that a general high order density correlation function reduces to the product of pair density-density correlations.

The paper is organized as follows. In Sec. II, we introduce the action for the $(1 + 3)$-level dipolar gas interacting with quantum electromagnetic field. An effective action for the light propagating in the gas, is derived in Sec. III. We evaluate the propagator for the electric field and identify the localization length of light in the gas in Sec. IV. There we also present simple relationship between the density-density correlation function and the light propagator, and analyze the result for various correlations in the gas. Finally, in Sec. V we conclude.

Throughout the paper we adopt the system of units in which the speed of light c, the electron charge e, and the Planck constant $\hbar$ are set to unity, $c = e = \hbar = 1$.

II. FIELD THEORY DESCRIPTION OF THE SYSTEM

A. General considerations

The propagation of light in a medium is described by the Maxwell equations supplemented with ones characterizing the properties of the medium. We express the solution in terms of propagators that relate the amplitude of the electromagnetic 4-vector potential $A_k(X)$ ($k = 0, 1, 2, 3$) at a space-time position $X = t, \mathbf{x}$, with $A_n(X')$ and is defined as

$$D_{kn}(X - X') = \langle G|TA_k(X)A_n(X')|G\rangle \tag{1}$$

where $|G\rangle$ is the ground state of the system, when the all atoms in the ground states and the photons are absent. We have used above the fact that the medium is transactionally invariant. For the medium that consists of scatters at random positions, the propagator of a given sample depends on the location of scatterers. To get general characteristics for light propagation in the disorder medium one have to average the propagator over disorder:

$$\bar{D}_{kn}(X - X') = \langle\langle G|TA_k(X)A_n(X')|G\rangle\rangle_{dis} \tag{2}$$

The averaged propagator is referred to as the amplitude propagators.

For further calculation the quantum field theory methods are applied. Thus, for the evaluation the expression (2) we have to construct the action for the light interacting with dipole gas. The action for the system depends on atoms degrees of freedom as well as electromagnetic field one. In a general form it can be written as

$$S = S(A_k, \psi_i^n, \bar{\psi}_i^n) \tag{3}$$

where ψ_i^n are fields corresponds to atoms and A_k 4-vector of electromagnetic field. Therefore the partition function Z for the system is

$$Z = \int \prod_{k=0,1,2,3} \mathcal{D}A_k \prod \mathcal{D}\left\{\psi_i^n, \bar{\psi}_i^n\right\} \exp\left[iS(A_k, \psi_i^n, \bar{\psi}_i^n)\right] \tag{4}$$

On the base of the participation function (4) we can evaluate the properties of a certain realization of disorder. To find the averaged propagators (2) one have averaged over disorder. Applying continual approach we figure distribution for realization of the number density $\rho(x)$ as:

$$P[\rho(x,t)] = C\exp\left[-\sum^n \int dx_1 \ldots dx_n \frac{\rho(x_1 t_1)\rho(x_2 t_2) \cdot \ldots \cdot \rho(x_n t_n)}{n!\Gamma^{(n)}(x_1 \ldots x_n, t_1 \ldots t_n)}\right] \tag{5}$$

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Here $\Gamma^{(n)}(x_1 \ldots x_n, t_1 \ldots t_n)$ is n-cumulant for the distribution. The first cumulant is equal to the averaged number density $\rho_0(x,t)$, while the second one coincides with dispersion for the number density :

$$\Gamma^{(2)}(x_1 t_1, x_2 t_2) = \langle (\rho(x_1 t_1) - \rho_0(x_1 t_1))(\rho(x_2 t_2) - \rho_0(x_2 t_2)) \rangle \tag{6}$$

If the number density of scatters is not high, the other cumulant are close to zero. For further evaluation we take into account that the number density can evolve with time, then the improved distribution (5) for $\rho(x,t)$ can be depicted as:

$$P[\rho(x,t)] \sim \exp\left[-\frac{1}{2} \int dx dx' \frac{(\rho(x,t) - \rho_0(x,t))\,(\rho(x',t') - \rho_0(x',t'))}{\Gamma^{(2)}(x,t;x',t')} \right] \tag{7}$$

There the averaging-out over disorder realization results in appearing a supplementary field $\rho(x,t)$

$$Z = \int \prod_{k=0,1,2,3} \mathcal{D} A_k \prod \mathcal{D}\left\{ \psi_i^n, \bar{\psi}_i^n \right\} \mathcal{D}\rho(x,t)\, P[\rho(x,t)] \exp\left[iS(A_k, \psi_i^n, \bar{\psi}_i^n) \right] \tag{8}$$

B. Model for atoms.

For the evaluation the field propagator we are in need of the full action for the system $S(A_k, \psi_i^n, \bar{\psi}_i^n)$. We start the analysis from consideration of a single atom immersed into an external electromagnetic field. The total action for a single atom S_{tot_atom} in the electromagnetic field consists of three terms: the action of the free atom S_{atom}, the free electromagnetic field S_f and the term caused by the interaction of the electromagnetic field with the atom S_{int}.

Let us find the action of a free atom. All evaluation will be carried out in the assumptions, that the atom is infinitively heavy. It allows us to take into account the evolution of just internal degrees of freedom. The translation degrees of freedom of the atom are not taken into account, because the effect of the momentum exchange between the atom and incident electromagnetic field is supposed to be small. In general case atomic structure is fairly complex and with increasing the number of electrons in the atom its complexity is increasing and the number of energy states grows [17, 18]. The action of the external electromagnetic field inspires transitions between the atomic states. If the atom with ground state energy ε_g and excited state energy ε_a is subjected to the monochromatic radiation with frequency

ω and the conditions $\omega \approx |\varepsilon_a - \varepsilon_g|$ holds true, the most of transitions occurs between the ground and the excited state. The others states does not affect on the atom dynamics in the electromagnetic field. It is assumed the character of the resonance transitions to dipole. In accordance with the selection rules for the dipole transitions, the final orbital momentum for an atom L_f and initial orbital momentum L_i are related by $|L_i - L_f| = 1$. Thus, if the ground state is singlet, the excited state has to be triplet, then transitions takes between four states $|E, l\rangle$ with the energy E and projection of orbital momentum l

$$
\begin{aligned}
|g\rangle &= |\varepsilon_g, l = 0\rangle \\
|-\rangle &= |\varepsilon_a, l = -1\rangle \\
|0\rangle &= |\varepsilon_a, l = 0\rangle \\
|+\rangle &= |\varepsilon_a, l = +1\rangle
\end{aligned}
\tag{9}
$$

So consider the system that contains atoms in the lowest $(E = \varepsilon_g, l = 0)$ and first excited states $(E = \varepsilon_a, l = -1, 0, 1)$ where we let c_i^+ with $i = g, -, 0, +$: create these respective states. If $|Vac\rangle$ is an atom vacuum state (i.e. the state that corresponds to the atom absence), these operators satisfy the following condition:

$$
\begin{aligned}
|i\rangle = c_i^+|Vac\rangle \qquad |Vac\rangle = c_i|i\rangle \qquad i = -1, 0, 1, g \\
\{c_i^+, c_k\} = \delta_{ik} \qquad \{c_i^+, c_k^+\} = 0, \qquad \{c_i, c_k\} = 0, \ \forall i, k
\end{aligned}
\tag{10}
$$

We will use further the change of basis $c_g^+, c_x^+ = \left(c_-^+ + c_+^+\right)/\sqrt{2}, c_y = -i\left(c_+^+ - c_-^+\right)/\sqrt{2}, c_z^+ = c_0^+$, and the hamiltonian of a free atom can be described in new terms of annihilation and creation operators c_i, c_i^+ where $i = g, x, y, z$ as

$$
\hat{H} = \varepsilon_a \hat{c}_x^+ \hat{c}_x + \varepsilon_a \hat{c}_y^+ \hat{c}_y + \varepsilon_a \hat{c}_z^+ \hat{c}_z + \varepsilon_g \hat{c}_g^+ \hat{c}_g
\tag{11}
$$

Let us introduce the Grassmann fields ψ_i and $\bar{\psi}_i$ that correspond to annihilation and creation operators c_i, c_i^+, $i = g, x, y, z$. If the fields ψ_i and $\bar{\psi}_i$ correspond to coordinate and momentum, then the free atom Lagrangian takes the form:

$$
L_{atom} = \sum_{i=g,x,y,z} \bar{\psi}_i(t) i\partial_t \psi_i(t) - \varepsilon_g \bar{\psi}_g(t)\psi_g(t) - \varepsilon_a \sum_{\alpha=x,y,z} \bar{\psi}_a(t)\psi_a(t)
\tag{12}
$$

and the action for a single atom

$$
S_{atom} = \int dt \left[\sum_{i=g,x,y,z} \bar{\psi}_i(t) i\partial_t \psi_i(t) - \varepsilon_g \bar{\psi}_g(t)\psi_g(t) - \varepsilon_a \sum_{\alpha=x,y,z} \bar{\psi}_a(t)\psi_a(t) + i\delta \sum_{i=g,x,y,z} \bar{\psi}_i(t)\psi_i(t) \right]
\tag{13}
$$

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A short imaginary parts appears to satisfy causality principle for atom evolution.Taking the energy of the ground state is equal to zero we can rewrite action for a single atom as

$$S_{atom} = \int \bar{\psi}_g(t) \left[i\partial_t + i\delta \right] \psi_g(t)dt + \int \bar{\psi}_\alpha(t) \left[i\partial_t - \Delta + i\delta \right] \psi_\alpha(t)dt, \Delta = \varepsilon_a - \varepsilon_g \qquad (14)$$

C. Interaction of atoms with light

The other two terms of the total action (S_{tot_atom}) S_f and S_{int} depend on the properties of electromagnetic field. If the electromagnetic field is determined by 4-potential A_i, the action associated with the free e-m field[19, 20] is

$$S_f = -\frac{1}{4\pi c} \int d^4x \left(\frac{1}{4} F_{ik} F^{ik} + \frac{1}{2\xi} (\partial^k A_k)^2 \right) \qquad F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \qquad (15)$$

The constant ξ appears as result of gauge invariance for the electromagnetic fields[19, 20]. Further we choose an appropriate gauge for evaluations. The term resulted from the interaction has to consists of field variable A_i as well as 4-current j^i, which is determined by atom properties:

$$S_{int} = -\frac{1}{c^2} \int d^4x A_i j^i \qquad (16)$$

If the atom is placed at the point $\mathbf{x_n}$ and the wavelength of electromagnetic field surpasses remarkably its size, we can treat the atom as point dipole and the 4-current produced by it is

$$j^n(xt) = \left\{ \begin{array}{c} c\rho(xt) \\ \vec{j}(xt) \end{array} \right\} = \left\{ \begin{array}{c} -cd_i(t)\partial_i\delta(\vec{x} - \vec{x}_a) \\ \dot{d}_i(t)\delta(\vec{x} - \vec{x}_a) \end{array} \right\} \qquad (17)$$

Integrating expression (16) over time and applying for the 4-current (17), we get :

$$S_{int}^n = -\frac{1}{c^2} \int d^4x A_i j^i = \frac{1}{c} \int E_\alpha(x_n, t)d_\alpha(t)dt \qquad \alpha = 1, 2, 3 \qquad (18)$$

where $E_\alpha(x, t)$ is electric field strength:

$$E_\alpha(x, t) = - \left[\partial_\alpha A_0(x, t) + \partial_0 A_\alpha(x, t) \right] \qquad (19)$$

If d_0 is the value of dipole moment associated with transition $\varepsilon_g \rightarrow \varepsilon_a$, the component of the operator $d_\alpha(t)$ is written as

$$d_\alpha(t) = d_0 \left[\bar{\psi}_\alpha(t)\psi_g(t) + \bar{\psi}_g(t)\psi_\alpha(t) \right]$$

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The expression (18) is exact for point-like dipole.

Thus the total action for the system of the atom and the electromagnetic field:

$$
\begin{aligned}
S_{tot_atom} &= -\tfrac{1}{4\pi c} \int d^4x \left(\tfrac{1}{4} F_{ik} F^{ik} + \tfrac{1}{2\xi}(\partial^k A_k)^2 \right) \\
&\quad \tfrac{1}{c} \int dt E_\alpha(x_a, t) d_0 \left[\bar\psi_\alpha(t)\psi_g(t) + \bar\psi_g(t)\psi_\alpha(t) \right] \\
&\quad \int dt \bar\psi_g(t) \left[i\partial_t + i\delta \right] \psi_g(t) dt + \int \bar\psi_\alpha(t) \left[i\partial_t - \Delta + i\delta \right] \psi_\alpha(t)
\end{aligned}
\tag{20}
$$

The total action of the atoms ensemble S_{total} and the electromagnetic field can be found by the generalizing (20). Naively one may assume the total action for an atomic ensemble is sums actions of free atoms and interaction terms over all ensemble, and the free electromagnetic field term. Applying expression for S^n_{atom} the action of a single atom at a point x_n (14) and its current $j^i(\mathbf{x_n})$ (17):

$$
S'_{total} = \sum_n S^n_{atom} - \frac{1}{c} \int A_i \sum_n j^i(\mathbf{x_n}) + S_f(A_i)
\tag{21}
$$

But this naive approach leaves out of account the interactions between atoms, resulted in an additional term in the total action. The force acted between two atoms is referred to as the Van der Waals force. The elementary processes on which the Van der Waals interaction is based is the exchange of the pair photons between the two atoms. This force depends on the distance as well as energy of resonance transitions. If the distance between the dipoles R is short, the virtual photons propagate instantly and the force is proportional to R^{-7}. In the opposite case, the delay in propagation of photons should be taken into account. The character dependence force on the distance is changed and has a form R^{-8}. The interaction between dipoles has electromagnetic nature. Thus the electromagnetic filed in the dipole gas consists of two parts: the first one originates from the external source A_i^{ext} and the second part appears as a result of interaction between dipoles A_i^{qn}. Further we are refereed to A_i^{ext}, A_i^{qn} as "external" and "quantum" fields. Therefore the total action for dipole gas in the electromagnetic field takes the form:

$$
S_{total} = \sum_n S^n_{atom} - \frac{1}{c} \int \left(A_i^{ext} + A_i^{qn} \right) \sum_n j^i(\mathbf{x_n}) + S_f(A_i^{qn}) + S_f(A_i^{ext})
\tag{22}
$$

If the external field is equal to zero, one can integrates over the quantum field A_q and the new effective action is determined by the atomic freedom of degree. This action includes sum free atomic action for all atoms 13 and the term S_{AA} related with interquartile interaction

$$
S_{AA} = -\frac{1}{c^3} \int d^4x d^4x' \frac{J_m(x) J^m(x')}{|\overline{x} - \overline{x'}|} \delta(\tau - \tau' - |\overline{x} - \overline{x'}|)
\tag{23}
$$

where $J^m = \sum_n j^m(\mathbf{x_n})$ is the total 4-current caused by all dipoles. If we consider the limit case, when the interaction can be treated as instant, i.e. $\Delta \cdot R_{mn}/c \ll 1$ (R_{mn} is distance between atoms n and m) the interaction takes the form:

$$S_{AA} = -\frac{1}{2} \sum_{m,n,m\neq n} \int H_{mn}(t)dt$$

$$H_{mn}(t) = \frac{d_\alpha^n(t)\, d_\alpha^n(t)\, R_{mn}^2 - 3d_\alpha^n(t)\, d_\beta^m(t)\, R_{\alpha,mn}R_{\beta,mn}}{R_{mn}^5}$$

i.e. we get the dipole-dipole interaction for static limit, as we should. The total action S_{total} for the dipole gas is given by sum22 with $j^i(\mathbf{x_n})$ determined by 17.

Because the ground state of the system can be determined by as product of the atomic the ensemble ground state and the electric field ground state $|0\rangle$ (the state when the number of photons is equal to 0), the ground of the system is

$$|G\rangle = |0\rangle \prod_n |g\rangle_n = |0\rangle \prod_n \bar{\psi}_g |Vac\rangle_n$$

the partition function for the ensemble is

$$Z = \lim_{t_i \to -\infty} \lim_{t_f \to \infty} \mathcal{N} \int \mathcal{D}\left\{ A^{ext,0} A^{ext,1} A^{ext,2} A^{ext,3} \right\} \mathcal{D}\left\{ A^{qn,0} A^{qn,1} A^{qn,2} A^{qn,3} \right\}$$
$$\cdot \prod_n \prod_{i=g,1,2,3} \mathcal{D}\psi_i^n \mathcal{D}\bar{\psi}_i^n \psi_g^n(t_f)\, \bar{\psi}_g^n(t_i) \cdot \exp[iS_{total}]$$

here $\mathcal{N}$ is fields independent normalization constant.

III.   EFFECTIVE FIELD THEORY FOR LIGHT

Above we have found the partition function for the interacting atomic ensemble. The paper object is to derive the Greens function for electromagnetic field. To carry out this deducing, we have to integrate over atomic fields ψ_i^n. The integration over atomic variables in the partitions function for the interacting dipole gas can not be performed accurately and some approximations must be employed. For weak interaction, perturbation theory can be applied, when the interacting term $\exp(iS_{ints})$ is expanded in the form of a power series:

$$\exp(iS_{ints}) = 1 - \frac{S_{int}^{n2}}{2} + \frac{S_{int}^{n4}}{24} + \dots$$

we keep the terms up to the forth order. For brevity it is convenient to introduce the following notation for the fields:

$$A^i = \frac{(A^{qn})^i + (A^{ext})^i}{\sqrt{2}}$$

$$B^i = \frac{(A^{qn})^i - (A^{ext})^i}{\sqrt{2}}$$

Then the partition function for the dipole gas is

$$Z = Z_1 \cdot Z_2$$

$$Z_n = \lim_{t_i \to -\infty} \lim_{t_f \to \infty} \mathcal{N}_n \int \prod_{i=g,1,2,3} \left[\mathcal{D}\psi_i^n \mathcal{D}\bar{\psi}_i^n \right] \psi_g^n(t_f) \bar{\psi}_g^n(t_i) \left(1 - \frac{S_{int}^{n2}}{2} + \frac{S_{int}^{n4}}{24} \right) \cdot \exp[iS_{atom}^n]$$

$$Z_1 = \mathcal{N}_A \int \mathcal{D}\left\{ A^0 A^1 A^2 A^3 \right\} \exp[iS_{field}^1] \prod_n Z_n$$

$$Z_2 = \mathcal{N}_B \int \mathcal{D}\left\{ B^0 B^1 B^2 B^3 \right\} \exp[iS_{field}^2]$$

where

$$
\begin{array}{ll}
S_{field}^1 = \frac{1}{4\pi c} \int d^4 x \left[\frac{1}{4} F_{im} F^{im} + \frac{1}{2\xi}(\partial^k A_k)^2 \right] & F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \\
S_{field}^2 = \frac{1}{4\pi c} \int d^4 x \left[\frac{1}{4} P_{im} P^{im} + \frac{1}{2\xi}(\partial^k B_k)^2 \right] & P_{ik} = \frac{\partial B_k}{\partial x^i} - \frac{\partial B_i}{\partial x^k} \\
S_{int}^n = \frac{\sqrt{2}d_0}{c} \int d\tau \cdot E_\alpha(x\tau) \left[\bar{\psi}_\alpha^n(\tau)\psi_g^n(\tau) + \bar{\psi}_g^n(\tau)\psi_\alpha^n(\tau) \right] & \\
S_{atom}^n = \int d\tau \left[\sum_{\alpha=1}^{3} \bar{\psi}_\alpha^n(\tau)(i\partial_\tau - \Delta + i\delta)\psi_\alpha^n(\tau) + \bar{\psi}_g^n(\tau)(i\partial_\tau - i\delta)\psi_g^n(\tau) \right] & \\
E_\alpha(x_n, \tau) = -(\partial_\alpha A_0(x_n, \tau) + \partial_0 A_\alpha(x_n, \tau)) &
\end{array}
\tag{24}
$$

and $\mathcal{N}_A, \mathcal{N}_B, \mathcal{N}_n$ are constants. Because the atomic fields do not mix in full action, the integration for each atom can be performed independently. As result of integration we get finally for Z_n

$$
Z_n = \breve{\mathcal{N}}_n \left(
\begin{array}{c}
1 - i\frac{2d_0^2}{c^2} \int dt_1 dt_2 E_a(x_n, t_1) E_a(x_n, t_2) \cdot G_e(t_1 - t_2) \\
-i\left(\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot G_e(t_1 - t_2) G_g(t_2 - t_3) G_e(t_3 - t_4) \\
\cdot E_a(x_n, t_1) E_a(x_n, t_2) E_b(x_n, t_3) E_b(x_n, t_4)
\end{array}
\right)
\tag{25}
$$

where the atomic Greens function for the ground $G_g(t_2 - t_1)$ and the excited states $G_e(t_2 - t_1)$ are defined as:

$$\langle \psi_g^n(t_2)\bar{\psi}_g^n(t_1) \rangle = iG_g(t_2 - t_1) = \theta(t_2 - t_1)$$

$$\langle \psi_a^n(t_2)\bar{\psi}_b^n(t_1) \rangle = i\delta_{ab}G_e(t_2 - t_1) = \theta(t_2 - t_1) \exp\left[-i\Delta(t_2 - t_1)\right]\delta_{ab}$$

The final expression for $\prod_n Z_n$ up to the forth order for d_0 is

$$
\prod_n Z_n = \breve{\mathcal{N}}
\begin{pmatrix}
1 - \sum_n i\frac{2d_0^2}{c^2} \int dt_1 dt_2 E_a(x_n,t_1) E_a(x_n,t_2) \cdot G_e(t_1-t_2) \\[2mm]
+\frac{1}{2} \sum_{n,m,n\neq m} \left(-i\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot G_e(t_1-t_2) G_e(t_3-t_4) \\[2mm]
E_a(x_n,t_1) E_a(x_n,t_2) E_b(x_m,t_3) E_b(x_m,t_4) \\[2mm]
-\sum_n i\left(\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot G_e(t_1-t_2) G_g(t_2-t_3) G_e(t_3-t_4) \\[2mm]
\cdot E_a(x_n,t_1) E_a(x_n,t_2) E_b(x_n,t_3) E_b(x_n,t_4)
\end{pmatrix}
\tag{26}
$$

If we introduce the atom number density as

$$
\rho(x) = \sum_n \delta(x-x_n)
$$

The discrete description of atomic ensemble can be replaced by continuous one and equation 26 transforms into

$$
\prod_n Z_n = \breve{\mathcal{N}}
\begin{pmatrix}
1 - i\frac{2d_0^2}{c^2} \int dt_1 dt_2 d^3x \cdot \rho(x) E_a(x,t_1) E_a(x,t_2) \cdot G_e(t_1-t_2) \\[2mm]
+\frac{1}{2}\left(-i\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot d^3x d^3y \cdot \left[\rho(x)\rho(y) - \rho(x)\delta^3(x-y)\right] \\[2mm]
\cdot G_e(t_1-t_2) G_e(t_3-t_4) E_a(x,t_1) E_a(x,t_2) E_b(y,t_3) E_b(y,t_4) \\[2mm]
-i\left(\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 d^3x \cdot \rho(x) \cdot G_e(t_1-t_2) G_g(t_2-t_3) G_e(t_3-t_4) \\[2mm]
\cdot E_a(x,t_1) E_a(x,t_2) E_b(x,t_3) E_b(x,t_4)
\end{pmatrix}
\tag{27}
$$

The dipole density $\rho(x)$ is stochastic value and depends on realization of disorder in the investigated sample. Unless the problem under consideration is light propagation in a certain sample, the answer has to averaged over density realization. Denote by $\mathbf{P}\left[\rho(x)\right]$ the probability density that the atom number density is equal to $\rho(x)$. The rigorous definition of this density reads

$$
\mathbf{P}\left[\rho(x)\right] = C \exp\left[L\left[\rho(x)\right]\right]
$$

where the normalization constant is defined as

$$
C^{-1} = \int D\rho(x) \exp\left[L\left[\rho(x)\right]\right]
$$

Generally, realization of disorder may vary with time (it means, in the course of evolution the corresponding cumulants change their values), but we restrict our investigation by the

frozen disorder, where the first and second cumulant are not equal to zero. This approximation holds true for low density, when a typical contributions of the higher cumulants drops remarkably with increasing its order[21], so $L[\rho(x)]$ can expressed as

$$L[\rho(x)] = -\frac{1}{2}\int dx_1 dx_2 \frac{(\rho(x_1) - \rho_0(x_1))(\rho(x_2) - \rho_0(x_2))}{\Gamma^{(2)}(x_1, x_2)}$$

Moreover, the atomic gas is assumed to be translation invariant and the density-density correlation can be expressed as

$$\Gamma^{(2)}(x, y) = \langle\rho(x)\rho(y)\rangle - \rho_0^2 = \rho_0^2 C(x - y) + \rho_0\delta^3(x - y)$$

$$\langle\rho(x)\rangle = \rho_0$$

where $C(x - y)$ is dimensionless correlation function for density,(density-density correlation is supposed to be independent on the atoms positions and interatomic distance affects on the the correlation function). So averaging over position we get eventually

$$\prod_n Z_n = \check{\mathcal{N}}\mathcal{K}(t, E, C, \rho_0) \tag{28}$$

$$= \check{\mathcal{N}}\begin{pmatrix} 1 - i\frac{2d_0^2\rho_0}{c^2}\int dt_1 dt_2 d^3x \cdot E_a(x, t_1)E_a(x, t_2) \cdot G_e(t_1 - t_2) \\ +\frac{1}{2}\left(-i\frac{2d_0^2\rho_0}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot d^3x d^3y \\ \cdot G_e(t_1 - t_2)G_e(t_3 - t_4)E_a(x, t_1)E_a(x, t_2)E_b(y, t_3)E_b(y, t_4) \\ +\frac{1}{2}\left(-i\frac{2d_0^2\rho_0}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot d^3x d^3y \cdot C(x - y) \\ \cdot G_e(t_1 - t_2)G_e(t_3 - t_4)E_a(x, t_1)E_a(x, t_2)E_b(y, t_3)E_b(y, t_4) \\ -i\left(\frac{2d_0^2}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 d^3x \cdot \rho_0 \cdot G_e(t_1 - t_2)G_g(t_2 - t_3)G_e(t_3 - t_4) \\ \cdot E_a(x, t_1)E_a(x, t_2)E_b(x, t_3)E_b(x, t_4) \end{pmatrix} \tag{29}$$

and the partition function is given by

$$Z_1 = \mathcal{N}\int \mathcal{D}\left\{A^0 A^1 A^2 A^3\right\}\exp[iS_{field}^1 + \log\mathcal{K}(t, E, C, \rho_0)]$$

IV. PROPAGATOR FOR THE ELECTRIC FIELD

In the previous section we have deduced the partition function for the atomic ensemble. On the basis of it we can establish a character of evolution for 4-th potential $A_i(xt)$, characterized the electromagnetic field evolution. In itself the 4-th potential is not physical

observable value, so we focus our attention on electric field and calculate the propagators for the electric field:

$$D_{ab}(R_1 t_1, R_2 t_2) = -i \langle E_a(R_1 t_1) E_b(R_2 t_2) \rangle$$

The propagators of Greens function carry out complete information about spectrum or dynamics of electric fields. It describes the propagation of waves form an external δ-delta function source. For further derivation it is convenient to express the action in terms of the electric field. Namely, the 4-vector A_α can be expressed in terms of 4-vector included components of electric field strength $E_\alpha(x\tau)$ supplemented with zero component $E_0(x\tau) = -\partial_0 A_0 - \partial_\alpha A_\alpha$. Hence, the action of free electromagnetic field15 in impulse-frequency representation converts into

$$S^1_{field} = \frac{1}{8\pi c} \int \frac{d^3 k}{(2\pi)^3} \frac{d\Omega}{2\pi} \left[-\frac{1}{\xi} E_0(k, \Omega) E_0(-k, -\Omega) + E_a(k\Omega) E_b(-k, -\Omega) \Pi_{ab}(k, \Omega) \right]$$

$$\Pi_{ab}(k, \Omega) = \frac{k_a k_b}{\Omega^2} + \delta_{ab} \frac{\Omega^2 - k_c^2}{\Omega^2}$$

The zero component of 4-vector (E_0, E_α), $\alpha = x, y, z$ corresponds to the calibration and does not influence on any physical observable values. This fact is proved by explicit representation of S^1_{field}, and $\mathcal{K}(t, E, C, \rho_0)$ does not contain E_0.

The Greens function of the photon state is then determined by

$$
\begin{aligned}
D_{ab}(R_1 t_1, R_2 t_2) &= -i \langle E_a(R_1, T_1) E_b(R_2 T_2) \rangle \\
&= -i \frac{\breve{\mathcal{N}} \int \prod_{k=0,1,2,3} \mathcal{D} E_k \exp\left[i S^1_{field}(E_k) \right] \mathcal{K}(E_k, t) \cdot E_a(R_1, T_1) E_b(R_2 T_2)}{\breve{\mathcal{N}} \int \prod_{k=0,1,2,3} \mathcal{D} E_k \exp\left[i S^1_{field}(E_k) \right] \mathcal{K}(E_k, t)}
\end{aligned}
$$

this formula can be represented diagrammatically(see fig 1).

The right hand side features the contributions of free evolution (diagram A), single scattering (diagram B), two scattering on two atoms (diagram A) and double scattering on the same atom (diagrams D-G). The explicit expression of the free photon propagator is

$$
\begin{aligned}
D^0_{ab}(R_1 T_1, R_2 T_2) &= -i \langle E_a(R_1, T_1) E_b(R_2 T_2) \rangle \\
&= \int \frac{d^3 k \, d\Omega}{(2\pi)^4} \exp\left[-i\Omega(T_2 - T_1) + i k_c (R_{2c} - R_{1c}) \right] \frac{4\pi c}{\Omega^2 - k^2 + i\varepsilon} \cdot (\Omega^2 \delta_{ab} - k_a k_b)
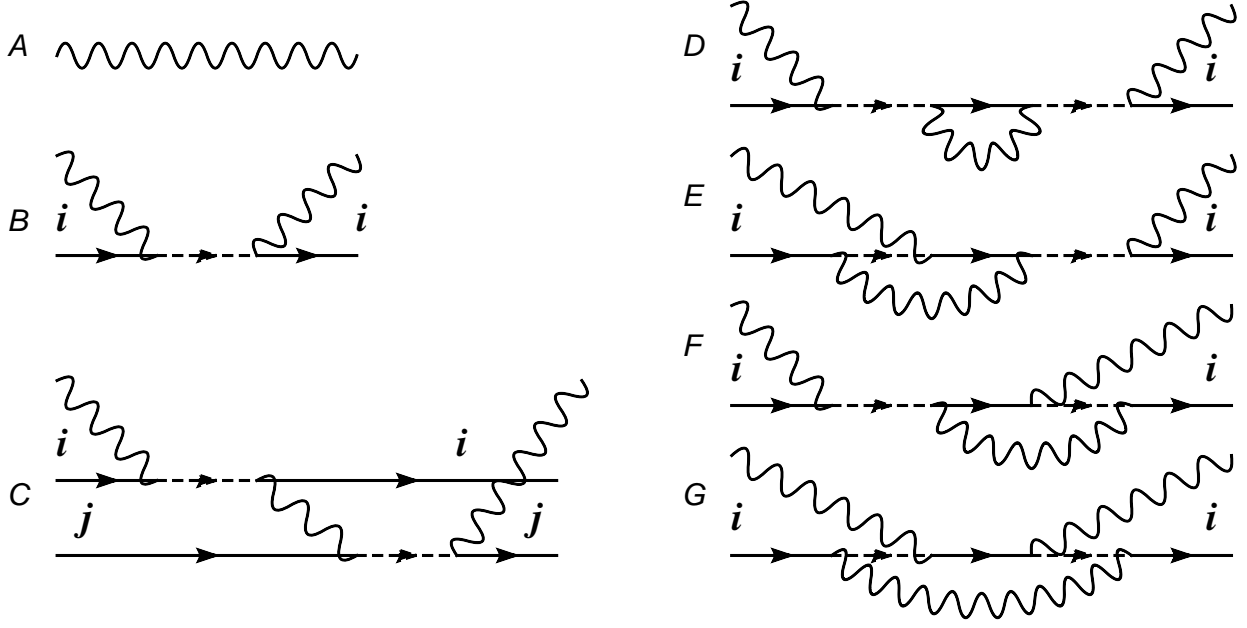\end{aligned}
$$

FIG. 1: Diagram representation for the photon Greens function. The wavy, solid and dash lines correspond to free photon propagator, the ground and excited atom Green functions respectively.

while the first order contribution is given by:

$$D_{ab}^1(R_1T_1,R_2T_2) = \frac{2d_0^2\rho_0}{c^2} \int dt_1 dt_2 d^3x \cdot [G_e(t_1 - t_2) + G_e(t_2 - t_1)]$$
$$\cdot D_{ap}^0(R_1T_1, xt_1)D_{pb}^0(xt_2, R_2T_2)$$

The second order term contains two different contributions. The first one appears as the second order Born term for the one-atom scattering and can be presented as:

$$D_{ab}^{2,1}(R_1T_1, R_2T_2) = i\left(\frac{2d_0^2}{c^2}\right)^2 \rho_0 \int dt_1 dt_2 dt_3 dt_4 d^3x$$

$$\cdot D_{pa}^0(R_1 - x, T_1 - t_1) D_{bq}^0(x - R_2, t_2 - T_2) \cdot \frac{D_{bb}^0(0, t_4 - t_3)}{3}$$

$$\begin{pmatrix} +G_e(t_4 - t_1)[G_g(t_1 - t_3)G_e(t_3 - t_2) + G_g(t_1 - t_2)G_e(t_2 - t_3)] \\ +G_e(t_1 - t_4)[G_g(t_4 - t_3)G_e(t_3 - t_2) + G_g(t_4 - t_2)G_e(t_2 - t_3)] \\ +G_e(t_4 - t_2)[G_g(t_2 - t_3)G_e(t_3 - t_1) + G_g(t_2 - t_1)G_e(t_1 - t_3)] \\ +G_e(t_2 - t_4)[G_g(t_4 - t_3)G_e(t_3 - t_1) + G_g(t_4 - t_1)G_e(t_1 - t_3)] \end{pmatrix}$$

The other term is in charge for the consequences photon scattering on two different atoms.So, in contrast to the previous the second order term it is proportional to the square of atom

density and is equal to

$$D_{ab}^{2,2}(R_1T_1, R_2T_2) = \left(\frac{2d_0^2\rho_0}{c^2}\right)^2 \int dt_1 dt_2 dt_3 dt_4 \cdot d^3x d^3y \cdot \left[1 + \frac{C(x-y) + C(y-x)}{2}\right]$$

$$\cdot \left(G_e\left(t_4 - t_1\right) + G_e\left(t_1 - t_4\right)\right)\left(G_e\left(t_2 - t_3\right) + G_e\left(t_3 - t_2\right)\right)$$

$$\cdot D_{pq}^0\left(x - y, t_4 - t_3\right) \cdot D_{ap}^0\left(R_1 - x, T_1 - t_1\right) D_{bq}^0\left(y - R_2, t_2 - T_2\right)$$

The straightforward result can be obtained in the momentum-frequency representation, where the free propagator is given by

$$D_{ab}^0\left(k, \omega\right) = \frac{4\pi c}{\omega^2 - k^2 + i\varepsilon} \cdot \left(\omega^2 \delta_{ab} - k_a k_b\right)$$

The terms $D_{ab}^{2,2}$ and D_{ab}^1 can be easily calculated and are equal to

$$D_{ab}^1\left(k, \Omega\right) = D_{ap}^0\left(k, \Omega\right) \cdot \chi\left(\Omega\right) \cdot D_{pb}^0\left(k, \Omega\right)$$

$$D_{ab}^{2,2}\left(k, \Omega\right) = D_{ap}^0\left(k, \Omega\right) \cdot \chi\left(\Omega\right)\left[D_{pq}^0\left(k, \Omega\right) + H_{pq}\left(k, \Omega\right)\right]\chi\left(\Omega\right) \cdot D_{qb}^0\left(k, \Omega\right)$$

$$\chi\left(\Omega\right) = \frac{4\Delta d_0^2 \rho_0}{c^2}\frac{1}{\Omega^2 - \Delta^2 + i\delta}$$

$$H_{ab}\left(k, \Omega\right) = \int \frac{d^3k'}{(2\pi)^3} D_{ab}\left(k', \Omega\right) \cdot \frac{C\left(k' - k\right) + C\left(k' + k\right)}{2}$$

The expression for $D_{ab}^{2,1}$ after the Fourier transformation takes the form

$$D_{ab}^{2,1}\left(k, \Omega\right) = i\left(\frac{2d_0^2}{c^2}\right)^2 \rho_0 B\left(\Omega\right) D_{ap}^0\left(k, \Omega\right) D_{pb}^0\left(k, \Omega\right)$$

$$B\left(\Omega_1\right) = \int \frac{d^3k}{(2\pi)^3}\frac{d\Omega}{2\pi}\frac{1}{3}D_{bb}^0\left(k, \Omega\right)\left[V\left(\Omega_1, \Omega\right) + V\left(-\Omega_1, \Omega\right)\right]$$

$$V\left(\Omega_1, \Omega\right) = G_g\left(\Omega_1 - \Omega\right)\left(G_e\left(-\Omega\right) + G_e\left(\Omega_1\right)\right)^2$$

The integral for $B\left(\Omega_1\right)$ is divergent and the singularities are connected to the large-k behavior of the Green's function in Fourier space. So, in order to remove the singularities, one can modify this behavior. The standard method, often used in quantum field theory, is to introduce a large-momentum cutoff. Because of the vector character of the photon propagator, we allow for two regularization procedures with different parameters, Λ_L and Λ_T

$$D_{aa}^0 (0, \Omega) = \int \frac{d^3k}{(2\pi)^3} D_{bb}^0 (k, \Omega)$$

$$= \int \frac{d^3k}{(2\pi)^3} \left[D_{bb}^0 (k, \Omega) - 4\pi c \left(1 - \frac{2\Omega^2}{k^2} \right) \right] + \int \frac{d^3k}{(2\pi)^3} 4\pi c \left(1 - \frac{2\Omega^2}{k^2} \right)$$

$$= c\Lambda_L^3 - c\Omega^2 \Lambda_T - i2c \left| \Omega^3 \right|$$

The employed regularization procedure is of course not unique, but it suffices for our purposes. The physical result is largely insensitive to such details. The final expression for $D_{ab}^{2,1} (k, \Omega)$

$$D_{ab}^{2,1} (k, \Omega) = i \left(\frac{2d_0^2}{c^2} \right)^2 \rho_0 B (\Omega) D_{ap}^0 (k, \Omega) D_{pb}^0 (k, \Omega)$$

$$B (\Omega) = -iC_1 \left[\frac{2\Delta}{\Omega^2 - \Delta^2 + i\delta} \right] - iC_2 \left[\frac{2\Delta}{\Omega^2 - \Delta^2 + i\delta} \right]^2 - c\frac{2 \left| \Omega^3 \right|}{3} \left[\frac{2\Delta}{\Omega^2 - \Delta^2 + i\delta} \right]^2$$

where the constant C_1 and C_2 are related with the cutoff parameter. The total propagator is given by:

$$D_{ab} (k, \Omega) = D_{ab}^0 (k, \Omega) + D_{ap}^0 (k, \Omega) \cdot \chi (\Omega) \cdot D_{pb}^0 (k, \Omega) \tag{30}$$

$$+ D_{ap}^0 (k, \Omega) \cdot \chi (\Omega) \left[D_{pq}^0 (k, \Omega) + H_{pq} (k, \Omega) \right] \chi (\Omega) \cdot D_{qb}^0 (k, \Omega)$$

$$+ D_{ap}^0 (k, \Omega) \chi (\Omega) \cdot \left(-ci\frac{2 \left| \Omega^3 \right|}{3\rho_0} \chi (\Omega) \delta_{pq} \right) \cdot D_{qb}^0 (k, \Omega)$$

$$\chi (\Omega) = \frac{4\Delta d_0^2 \rho_0}{c^2} \frac{1}{\Omega^2 - \Delta^2 + i\delta}$$

$$H_{ab} (k, \Omega) = \int \frac{d^3k'}{(2\pi)^3} D_{ab} (k', \Omega) \cdot \frac{C (k' - k) + C (k' + k)}{2} \tag{31}$$

We have omitted the terms included renormalization constants C_1, C_2, because they result in shifting of the values of dipole momentum d_0 and energy Δ.

To specify the problem completely one needs to give the value for the density-density correlation $C (r)$. We start consideration from white-noise correlation, where $C (r) = K\delta^3 (r)$. Such a function describes short-range correlation appears between the neighboring atoms. With accuracy of renormalization constant

$$H_{ab} (k, \Omega) = -2ic \left| \Omega^3 \right| K$$

and

$$D_{ab}(k, \Omega) = D_{ab}^0(k, \Omega) + D_{ap}^0(k, \Omega) \cdot \chi(\Omega) \cdot D_{pb}^0(k, \Omega)$$

$$+ D_{ap}^0(k, \Omega) \cdot \chi(\Omega) \cdot D_{pq}^0(k, \Omega) \cdot \chi(\Omega) \cdot D_{qb}^0(k, \Omega)$$

$$+ D_{ap}^0(k, \Omega) \chi(\Omega) \cdot \frac{-2ci|\Omega^3|}{3\rho_0}(1 + K\rho_0)\chi(\Omega) \cdot D_{pb}^0(k, \Omega)$$

$$\chi(\Omega) = \frac{4\Delta d_0^2 \rho_0}{c^2} \frac{1}{\Omega^2 - \Delta^2 + i\delta}$$

For the low density of scatters with the same accuracy the answer can be present in a compact form

$$D_{ab}(k, \Omega) = D_{ab}^0(k, \Omega) + D_{ap}^0(k, \Omega) \cdot \chi(\Omega) \cdot D_{pq}^0(k, \Omega) \cdot \chi(\Omega) \cdot D_{qb}^0(k, \Omega)$$

$$+ D_{ap}^0(k, \Omega) \cdot \frac{1}{[\chi(\Omega)]^{-1}}\left[1 - \frac{2ci|\Omega^3|(1 + K\rho_0)}{3\rho_0[\chi(\Omega)]^{-1}}\right] \cdot D_{pb}^0(k, \Omega)$$

$$\simeq D_{ab}^0(k, \Omega) + D_{ap}^0(k, \Omega) \cdot \tilde{\chi}(\Omega) \cdot D_{pb}^0(k, \Omega)$$

$$+ D_{ap}^0(k, \Omega) \cdot \tilde{\chi}(\Omega) \cdot D_{pq}^0(k, \Omega) \cdot \tilde{\chi}(\Omega) \cdot D_{qb}^0(k, \Omega)$$

$$\tilde{\chi}(\Omega) = \frac{4\Delta d_0^2 \rho_0}{c^2} \frac{1}{\Omega^2 - \Delta^2 + \frac{8i}{3c}\Delta d_0^2 |\Omega^3|(1 + K\rho_0)}$$

or

$$D_{ab}(\Omega, k) = 4\pi c \frac{\delta_{ab}\Omega^2(1 - \alpha(\Omega)) - k_a k_b}{\Omega^2(1 - \alpha(\Omega)) - k^2 + i\varepsilon} \frac{1}{(1 - \alpha(\Omega))}$$

$$\alpha(\Omega) = \frac{16\pi\Delta d_0^2 \rho_0}{c} \frac{1}{\Omega^2 - \Delta^2 + \frac{8id_0^2\Delta|\Omega^3|}{3c}(1 + K\rho_0)}$$

Here $\alpha(\Omega)$ coincides with permittivity for the dipole gas. Because of the correlation the imaginary part of permittivity has increased in contrast to the common dipolar gas. Such a behavior leads to increasing the scattering mean free path l_R. It is determined by the imaginary part of the pole

$$\frac{1}{2l_R} = \text{Im}\sqrt{\Omega^2(1 - \alpha(\Omega))}$$

The result reads

$$l_R \simeq \frac{2}{3\pi} \frac{|\Omega^2|(1 + K\rho_0)}{\rho_0} \frac{(\Omega^2 - \Delta^2)^2 + (\Gamma|\Omega^3|)^2}{(\Gamma|\Omega^3|)^2}$$

$$\Gamma = \frac{8d_0^2\Delta}{3c}(1 + K\rho_0)$$

17

As expected, this result indicates that with increasing correlation scattering is determined by multiply scattering, while for the low values K the imaginary part is determined by the evolution from the internal degrees.

Above we have analyzed the short-correlation case, but the situation becomes drastically different when long range correlation sets in due to induced particle aggregation, as for instance in gelly or aerogels. If it is the case the series30 can collected as follows:

$$
\begin{aligned}
D_{ab}\left(k,\Omega\right) &\simeq D_{ap}^{0}\left(k,\Omega\right)+D_{ap}^{0}\left(k,\Omega\right)\tilde{\chi}\left(\Omega\right)D_{pb}^{0}\left(k,\Omega\right) \\
&\quad + D_{ap}^{0}\left(k,\Omega\right)\cdot\tilde{\chi}\left(\Omega\right)\left[D_{pq}^{0}\left(k,\Omega\right)+H_{pq}\left(k,\Omega\right)\right]\tilde{\chi}\left(\Omega\right)\cdot D_{qb}^{0}\left(k,\Omega\right) \\
\tilde{\chi}\left(\Omega\right) &= \frac{4\Delta d_{0}^{2}\rho_{0}}{c^{2}}\frac{1}{\Omega^{2}-\Delta^{2}+\frac{8i}{3c}\Delta d_{0}^{2}\left|\Omega^{3}\right|}
\end{aligned}
\tag{32}
$$

Thus, with the accuracy upto the second order the electric field propagator can be rewritten as

$$
\begin{aligned}
D_{ab}\left(k,\Omega\right) &\simeq D_{ab}^{eff}\left(k,\Omega\right)+D_{ap}^{eff}\left(k,\Omega\right)\cdot\tilde{\chi}\left(\Omega\right)H_{pq}\left(k,\Omega\right)\tilde{\chi}\left(\Omega\right)\cdot D_{pb}^{eff}\left(k,\Omega\right) \\
D_{ab}^{eff}\left(k,\Omega\right) &= 4\pi c\frac{\delta_{ab}\Omega^{2}\left(1-4\pi c\tilde{\chi}\left(\Omega\right)\right)-k_{a}k_{b}}{\Omega^{2}\left(1-4\pi c\tilde{\chi}\left(\Omega\right)\right)-k^{2}+i\varepsilon}\frac{1}{\left(1-4\pi c\tilde{\chi}\left(\Omega\right)\right)}
\end{aligned}
\tag{33}
$$

The first term of the Green function can be attribute to a photon free propagation in the atomic resonance gas without correlation, while the second term is related with "scattering on density correlation" in such a medium. Taking into account, that the medium is translation invariant, one can decompose $H_{ab}\left(k,\Omega\right)$ as follows

$$
H_{ab}\left(k,\Omega\right) = \int\frac{d^{3}k'}{\left(2\pi\right)^{3}}D_{ab}\left(k',\Omega\right)\cdot\frac{C\left(k'-k\right)+C\left(k'+k\right)}{2} = \tilde{A}\left(k,\Omega\right)\delta_{ab}+\tilde{B}\left(k,\Omega\right)\frac{k_{a}k_{b}}{k^{2}}
\tag{34}
$$

Thus, in contrast to the short-range correlation, the long-range density correlations result in space anisotropy for the light propagation. The character of this anisotropy is determined by the coefficients $\tilde{A}\left(k,\Omega\right),\tilde{B}\left(k,\Omega\right)$. The function $H_{ab}\left(k,\Omega\right)$ can be rewritten as

$$
\begin{aligned}
H_{ab}\left(k,\Omega\right) &= -\int cC\left(R\right)\cos\left[\mathbf{kR}\right]\cdot\left(\delta_{ab}\Omega^{2}+\partial_{a}\partial_{b}\right)\frac{\exp\left[i\left|\Omega\right|R\right]}{R}\cdot d^{3}R \\
&= -\int cC\left(R\right)\cos\left[\mathbf{kR}\right]\cdot\left(\delta_{ab}\Omega^{2}+\delta_{ab}\frac{\Delta}{3}+\partial_{a}\partial_{b}-\delta_{ab}\frac{\Delta}{3}\right)\frac{\exp\left[i\left|\Omega\right|R\right]}{R}\cdot d^{3}R
\end{aligned}
$$

The tensor $H_{ab}(k, \Omega)$ can be decomposed into terms:

$$H_{ab}(k, \Omega) = H^1_{ab}(k, \Omega) + H^2_{ab}(k, \Omega)$$

$$H^1_{ab}(k, \Omega) = -\int cC(R)\cos[\mathbf{kR}] \cdot \left(\partial_a \partial_b - \delta_{ab}\frac{\Delta}{3}\right) \frac{\exp[i|\Omega|R]}{R} \cdot d^3R$$

$$H^2_{ab}(k, \Omega) = -\int cC(R)\cos[\mathbf{kR}] \cdot \delta_{ab}\left(\Omega^2 + \frac{\Delta}{3}\right) \frac{\exp[i|\Omega|R]}{R} \cdot d^3R$$

The trace for the first term is equal to 0, so a general expression for $H^1_{ab}(k, \Omega)$

$$H^1_{ab}(k, \Omega) = A(k, \Omega)\left(\delta_{ab} - 3\frac{k_a k_b}{k^2}\right)$$

and the value for $A(k, \Omega)$ can be found if we multiply the tensor on $\frac{k_a k_b}{k^2}$, so

$$\frac{k_a k_b}{k^2} H^1_{ab}(k, \Omega) = -2A(k, \Omega)$$

$$= -\frac{k_a k_b}{k^2}\int cC(R)\cos[\mathbf{kR}] \cdot \left(\frac{R_a R_b}{R^2} - \frac{\delta_{ab}}{3}\right) R$$

$$\cdot \frac{d}{dR}\left(\frac{1}{R}\frac{d}{dR}\left(\frac{\exp[i|\Omega|R]}{R}\right)\right) \cdot d^3R$$

$$= -\int cC(R)\cos[kR\cos\theta] \cdot \left(\cos^2\theta - \frac{1}{3}\right) R$$

$$\cdot \frac{d}{dR}\left(\frac{1}{R}\frac{d}{dR}\left(\frac{\exp[i|\Omega|R]}{R}\right)\right) \cdot 2\pi R^2 dR d\cos\theta$$

So after the simplification we get relationship for $A(k, \Omega)$

$$P(a) = \frac{2\sin a}{a}$$

$$A(k, \Omega) = -c\pi\int C(R)\left(P''(kR) + \frac{1}{3}P(kR)\right) \cdot \frac{d}{dR}\left(\frac{1}{R}\frac{d}{dR}\left(\frac{\exp[i|\Omega|R]}{R}\right)\right) \cdot R^3 dR$$

The evolution the second term is simpler, because it is proportional to δ_{ab}. After the replacement Laplacian we get

$$H^2_{ab}(k, \Omega) = -\delta_{ab}\int cC(R)\cos[\mathbf{kR}] \cdot \left(\frac{2\Omega^2}{3}\frac{\exp[i|\Omega|R]}{R} - \frac{4\pi}{3}\delta^3(R)\right) \cdot d^3R$$

$$= \delta_{ab}\frac{4\pi c}{3}C(0) - \delta_{ab}\frac{8\pi c\Omega^2}{3k}\int C(R) \cdot \exp[i|\Omega|R]\sin kR \cdot dR$$

The final expression for $H_{ab}(k,\Omega)$ is

$$H_{ab}(k,\Omega) = \left[A(k,\Omega) - B(k,\Omega) + \frac{4\pi c}{3} C(0) \right] \delta_{ab} - 3A(k,\Omega) \frac{k_a k_b}{k^2} \tag{35}$$

$$A(k,\Omega) = -c\pi \int C(R) \left(P''(kR) + \frac{1}{3} P(kR) \right) \cdot \frac{d}{dR} \left(\frac{1}{R} \frac{d}{dR} \left(\frac{\exp[i\,|\Omega|\,R]}{R} \right) \right) \cdot R^3 dR$$

$$B(k,\Omega) = \frac{8\pi c \Omega^2}{3k} \int C(R) \cdot \exp[i\,|\Omega|\,R] \sin kR \cdot dR$$

From the structure of the expression(35) one can conclude, the only long-range correlations cause space anisotropy for photon propagation.

V. CONCLUSIONS

We have studied the photon propagation through randomly distributed resonant atoms. Starting from the exact action included interatomic dynamics, atom-atom interaction(in electric-dipole approximation) and electric field atom interaction, we have derived the Green function for the dressed photon that describes both the inelastic processes like a spontaneous emission of the atoms and elastic scattering photon in gas with density-density correlations via resonant atom-field interaction. The proposed model of atoms allowed us to describe consequence of transitions in real atoms and abandoned two-level models [22] mostly used. The applied approach are not restricted to the rotating-wave approximation. We have shown that the energy associated with transition between resonance states does not change for low atomic density where the coupling parameter is $\frac{16\pi d_0^2 \rho_0}{\Delta}$ is small. There are two energy-transfer channels in general: one channel through resonant dipole-dipole interaction mediated by virtual-photon creation and destruction and the other one through emission and absorption of real photons. In particular for strong atom-field interaction, the (over a period averaged) energy transfer can be inhibited due to destructive interference of the two available transfer channels. The developed approach allows to consider both these channels simultaneously without separation. Short-range and long-range correlation for the atom densities have to be studied. From the general consideration, it was found that even for translation invariant medium, long-range correlation results to space anisotropy for the photon propagation. Clearly, the present analysis has left a number of open questions, on which future work will concentrate. In particular, the problem of energy transfer in ensemble of the moving atoms or influence of saturation effects on the character light propagation.

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